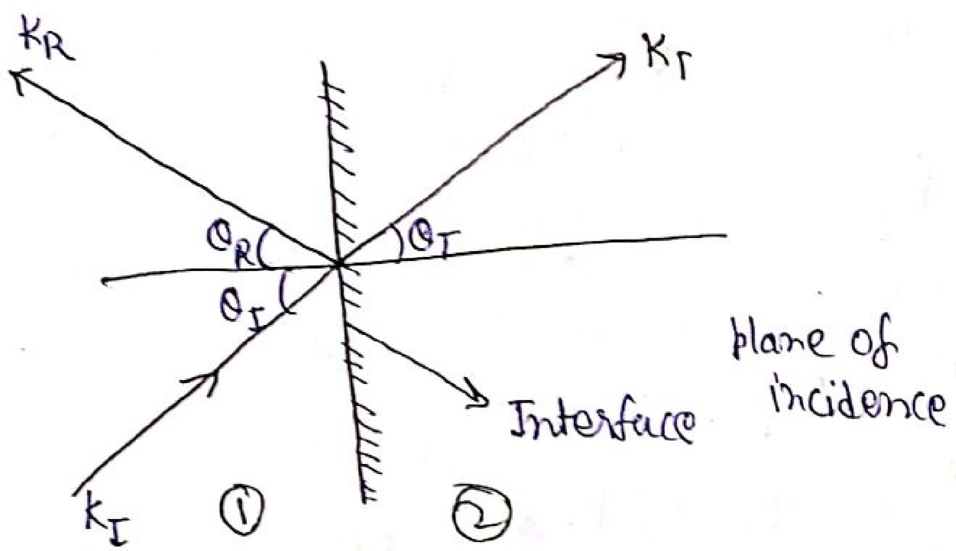
Reflection & Transmission at an oblique incidence:-



Let us, Consider a Case In which incident wave hits the boundary at an arbitrary angle θ_I. the fields of incident waves are:

$$\vec{E}_I(\vec{r},t) = \tilde{E}_{0I}\, e^{i(\hat{k}_I\cdot\vec{r}-\omega t)}$$

$$\vec{B}_I(\vec{r},t) = \frac{1}{v_1}(\hat{k}_I \times \vec{E}_I) \quad \text{———} \quad (1)$$

the fields for Reflected waves are.

$$\vec{E}_R(\vec{r},t) = \tilde{E}_{0R}\, e^{i(\hat{k}_R\cdot\vec{r}-\omega t)}$$

$$\vec{B}_R(\vec{r},t) = \frac{1}{v_1}(\hat{k}_R \times \vec{E}_R) \quad \text{———} \quad (2)$$

the fields for transmitted waves are:

$$\vec{E}_T(\vec{r},t) = \tilde{E}_{0T}\, e^{i(\hat{k}_T\cdot\vec{r}-\omega t)}$$

$$\vec{B}_T(\vec{r},t) = \frac{1}{v_2}(\hat{k}_T \times \vec{E}_T) \quad \text{———} \quad (3)$$

all the three waves have the same frequency ω and there wave no. vector or propogation Vector is related by:

$$k_I v_1 = k_R v_1 = k_T v_2 = \omega$$

Or, $$k_I = k_R = \frac{v_2}{v_1} k_T = \frac{n_1}{n_2} k_T \quad \text{---} (4)$$

Since, the tangential Comp. of e.f. must be Continuous across the boundry, $\underline{z=0}$, In that Case,

$$\vec{E}_I(z=0) + \vec{E}_R(z=0) = \vec{E}_T(z=0) \quad \text{---} (5)$$

using (5) it Can be write,

(i) $\omega_I = \omega_R = \omega_T = \omega$

(ii) $k_{Ix} = k_{Rx} = k_{Tx} = k_x$

(iii) $k_{Iy} = k_{Ry} = k_{Ty} = k_y$

Cond^ns (ii) & (iii) requires that the propogation Vectors k_I, k_R & k_T must lies in the plane of incidence, i.e.

First law:- $k_I \cdot \sin\theta_I = \underbrace{k_R \cdot \sin\theta_R}_{\text{Law of Reflection}} = k_T \cdot \sin\theta_T \quad \text{---} (6)$

θ_I is Called angle of incidence, θ_R is Called angle of Reflection & θ_T angle of transmittance

using eq^n (4), eq^n (6) Can be written as,

Second law:-

~~Second law~~

$$\boxed{\theta_I = \theta_R}$$ (This is known as 'Law of Reflection')

this is Called 'law of Reflection'. the angle of incidence is equal to angle of Reflection.

~~Second law:~~

AS for transmitted angle,

and,

third law:

$$\boxed{\frac{\sin\theta_T}{\sin\theta_I} = \frac{n_1}{n_2}} \quad \text{———— (8)}$$

This is 'Snell's law' or 'law of Refraction'. These are the three fundamental law of optics. Therefore any other waves can be expected to obey the same "optical laws" when they pass from one medium to another.

The boundary condns for this case are,

(i) $\epsilon_1(\vec{E}_{0I} + \vec{E}_{0R}) = \epsilon_2 \vec{E}_{0T}$

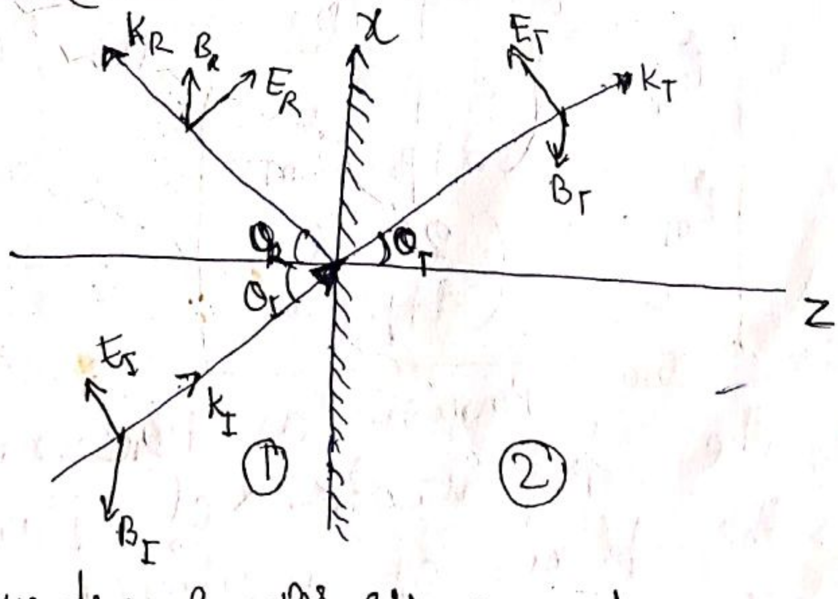
(ii) $(\vec{B}_{0I} + \vec{B}_{0R}) = \vec{B}_{0T}$

(iii) $(\vec{E}_{0I} + \vec{E}_{0R})_{x,y} = (\vec{E}_{0T})_{x,y}$ ———— (9)

(iv) $\frac{1}{\mu_1}(\vec{B}_{0I} + \vec{B}_{0R})_{x,y} = \frac{1}{\mu_2}(\vec{B}_{0T})_{x,y}$

(a) <u>parallel polarization</u>: —

Let us, consider, the polarization of incident wave is <u>parallel</u> to the plane of incidence. the reflected and transmitted waves are also polarized in this plane.



Using boundary condns (i), we get

$$\epsilon_1(-\vec{E}_{0I}\sin\theta_I + \vec{E}_{0R}\sin\theta_R) = \epsilon_2(-\vec{E}_{0T}\sin\theta_T) \quad \text{———— (10)}$$

Since, there will be no Comp. of m.f. in z-direction the boundary Condⁿ (ii) Can not be applied.

Condⁿˢ (iii) gives, —

$$\tilde{E}_{0I}\cos\theta_I + \tilde{E}_{0R}\cos\theta_R = \tilde{E}_{0T}\cos\theta_T \quad \text{——— (11)}$$

Condⁿˢ (iv) gives —

$$\frac{1}{\mu_1 v_1}(\tilde{E}_{0I} - \tilde{E}_{0R}) = \frac{1}{\mu_2 v_2}\tilde{E}_{0T}$$

$$\tilde{E}_{0I} - \tilde{E}_{0R} = \beta\,\tilde{E}_{0T} \quad \text{——— (12)}$$

wher, $\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$

Using the laws of Reflection & Refraction eqⁿ (11) becomes -

$$\tilde{E}_{0I} + \tilde{E}_{0R} = \alpha\,\tilde{E}_{0T} \quad \text{——— (13)}$$

$$\alpha = \frac{\cos\theta_T}{\cos\theta_I} \quad (\theta_I = \theta_R)$$

~~for the reflected and transmitted amplitude -~~

On Solving, eqⁿ (12) & (13), we get

$$\boxed{\begin{aligned} \tilde{E}_{0R} &= \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I} \\ \tilde{E}_{0T} &= \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I} \end{aligned}} \quad \text{——— (14)}$$

These are known as Fresnel's equations for the polarization of plane e.m. waves in the plane of incidence.

The transmitted wave is always in phase with the incident wave.

the Reflected wave is either 'in phase', if $\alpha > \beta$ or, 180° 'out of phase' if $\alpha < \beta$.

The amplitudes of the transmitted and reflected waves depend on the angle of incidence, bcoz α is a function of θ_I:

$$\alpha = \frac{\cos\theta_T}{\cos\theta_I} = \frac{\sqrt{1-\sin^2\theta_T}}{\cos\theta_I} = \frac{\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_I\right)^2}}{\cos\theta_I} \quad \text{---} (15)$$

Consider two cases:-

(i) for normal incidence i.e. $\theta_I = 0$

$\alpha = 1$

(ii) for grazing incidence i.e $\theta_I = 90^\circ$

'α' diverge i.e wave is totally reflected

and, when $\alpha = \beta$ in eq^n (14) there is no reflection and the incident angle at which this takes place is called Brewster's angle 'θ_B'.

~~There~~ at this angle the reflected wave is totally ~~transmitted~~ ~~& no~~ ~~reflected~~ extinguished.

$$\sin^2\theta_B = \frac{1-\beta^2}{\left(\frac{n_1}{n_2}\right)^2 - \beta^2}$$

if, $\mu_1 \simeq \mu_2$

$$\beta = \frac{n_1}{n_2}$$

$$\sin^2\theta_B = \frac{\beta^2}{1+\beta^2}$$

or

$$\boxed{\tan\theta_B = \frac{n_2}{n_1}} \quad \text{---} (16)$$

the incident intensity is:

$$I_I = \frac{1}{2}\epsilon_1 v_1 E_{0I}^2 \cos\theta_I$$

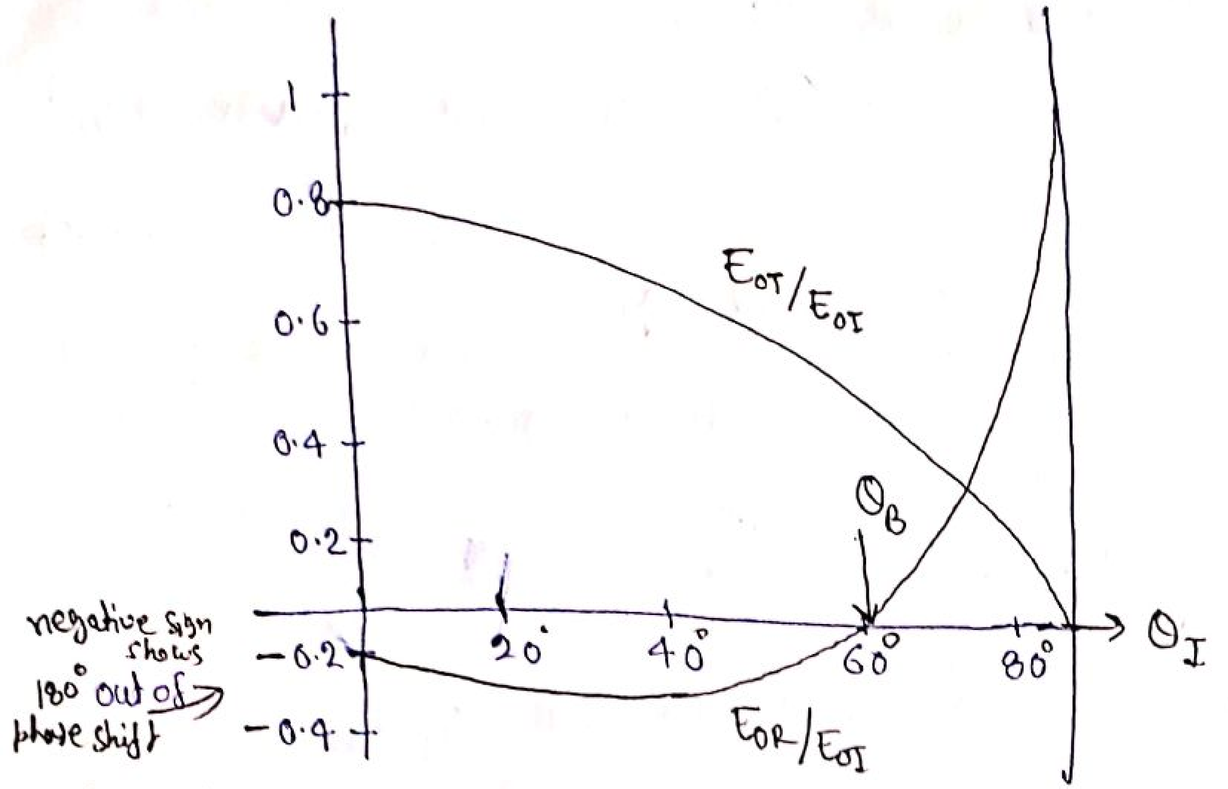


fig. shows, the plot of transmitted & reflected amplitudes as function of incident angle θ_I.

the reflected & transmitted intensities are:

$$I_R = \frac{1}{2}\epsilon_1 v_1 E_{0R}^2 \cos\theta_R'$$

$$I_T = \frac{1}{2}\epsilon_2 v_2 E_{0T}^2 \cos\theta_T$$

There fore, the <u>Reflection & transmission Coeff.</u> for waves polarized parallel to the planes are

$$R = \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2$$

$$T = \frac{I_T}{I_I} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left(\frac{E_{0T}}{E_{0I}}\right)^2 \cdot \frac{\cos\theta_T}{\cos\theta_I} \quad (17)$$

$$= \alpha\beta\left(\frac{2}{\alpha+\beta}\right)^2$$